

Modeling Compression Stress Distributions at the Interface Between Pallet Decks and Distribution Packaging

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April 3, 2012

Background

Unit loads are the form by which most industrial and consumer products are stored, shipped, and distributed. Exclusive of the product itself, the unit load portion of the supply chain consists of three basic components

Packaging

Pallets

Unit load handling equipment

The reaction of the products to the rigors of the movement through supply chains, from manufacturer to the customer, is a consequence of how these components mechanically interact. These mechanical interactions are both static and dynamic. The interface between the packaged product and the stresses exerted by the unit load handling, storage, and shipping devices is most often a pallet. Therefore, most of these dynamic and static stresses pass through the pallet prior to exposure of the package and product. When designing distribution packaging for unitized handling, storage, and shipping, the compression strength of the packaging becomes a major design criteria. Compression occurs when stacking of the packaged product on pallets and the subsequent stacking of unit loads, one on top of another, during storage and transportation. Often the highest compression stress occurs when unit loads are stack stored in warehouses or

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distribution centers. Under these conditions, the compression strength to which we design rigid packaging such as corrugated containers, plastic pails, drums, bottles, etc., becomes a function of the bearing area between the pallet deck and the packaging and the applied force. With a wood pallet, it is widely recognized that the bearing area is a function of deck design and specifically spacing between deckboards of the top and bottom decks of the pallet. What is not well understood is how the stiffness of the pallet decks affects the bearing area and consequently the compression stress on the packaging and its contents. The potential for non-uniform stress distributions and stress concentrations at the interface between the pallet deck and packaged product is shown schematically in Figure 1 and is associated with deflection of the pallet deck. Han et al. (2007) and Yoo (2008) showed that the pallet deck deforms under load and this reduces significantly the effective bearing area. Using topographical mapping techniques and FEA modeling, they predicted stress amplification factors of five to six times the average applied stress. The research reported here is the development of a generalized, closed form, mathematical model that more accurately predicts the compression stress distribution at the interface between packaging and the pallet deck, as a function of packaging stiffness and pallet deck stiffness. It has been discovered, that with some modification, the engineering principles of an elastic beam supported by an elastic foundation, predictions of compression stress distributions at the interface are possible Yoo (2011).

Model Development

To apply the principles of a beam supported by a deformable elastic foundation to the interface between a pallet deck and distribution packaging, the beam shall represent the pallet deck and the foundation shall represent the packaging and its contents. As shown in Figure 2, the actual situation within a unit load has been inverted for this application. The pallet section represents a deckboard spanning two stringers or block segments. General solutions for the beam deflection when supported on a deformable elastic foundation of stiffness, k , are shown in Figure 3.

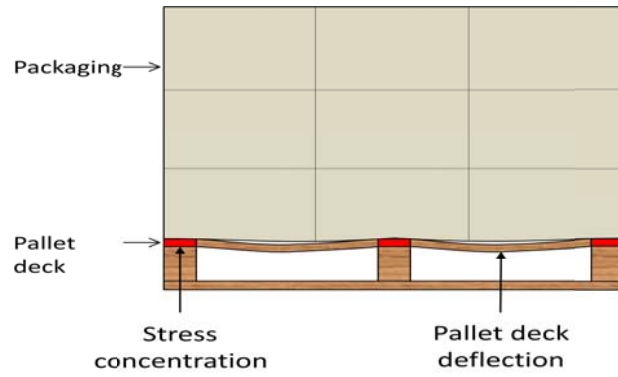


Figure 1. Schematic diagram showing locations of compression stress concentrations associated with deflection of a pallet deck within a unit load.

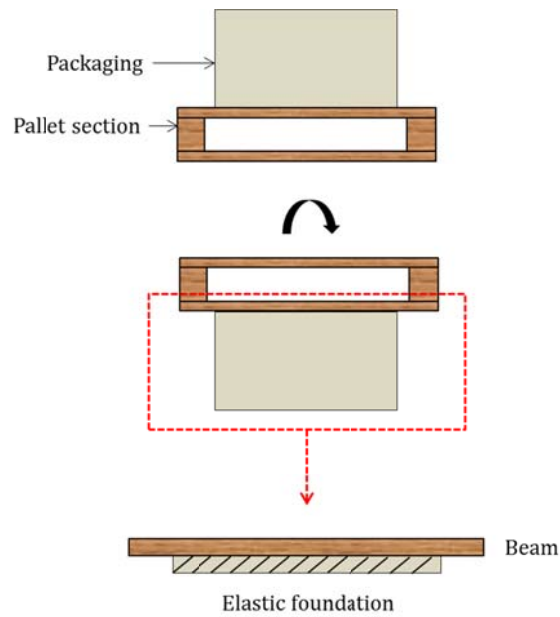
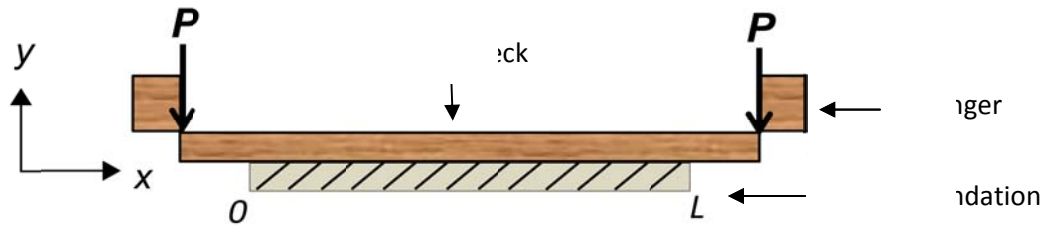


Figure 2. Conceptual application of a beam supported by a deformable elastic foundation to a package supported by a pallet deck.



General solution for beam deflection y at any point x

$$y(x) = y_o F_1(\lambda x) + \frac{1}{\lambda} \theta_o F_2(\lambda x) - \frac{1}{\lambda^2 EI} M_o F_3(\lambda x) + \frac{1}{\lambda^3 EI} Q_o F_4(\lambda x)$$

Where:

$$F_1(\lambda x) = \cosh \lambda x \cos \lambda x$$

$$F_2(\lambda x) = \frac{1}{2} (\cosh \lambda x \sin \lambda x + \sinh \lambda x \cos \lambda x),$$

$$F_3(\lambda x) = \frac{1}{2} (\sinh \lambda x),$$

$$F_4(\lambda x) = \frac{1}{4} (\cosh \lambda x \sin \lambda x - \sinh \lambda x \cos \lambda x),$$

$$\lambda = \sqrt{\frac{k}{4EI}}$$

k packaging stiffness

EI pallet deck stiffness

y_o deflection at $x = 0$

θ_o slope at $x = 0$

M_o moment at $x = 0$

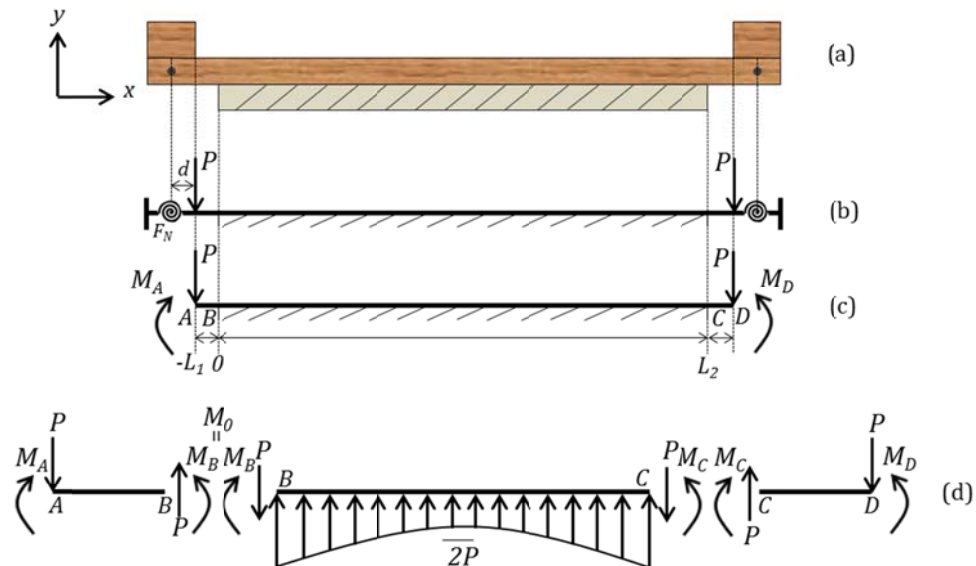
Q_o shear force at $x = 0$

Figure 3. Generalized solution for the deformation of an elastic beam (pallet deck supported by two stringers) supported by a uniformly distributed deformable elastic foundation (packaged product).

The force "P" is the reaction at the location of the pallet deck supports to the mass of the elastic foundation. Elastic response is assumed and has been confirmed by Weigel et al., 1999, at low levels of packaging and pallet deformation. However, theory assumes the ends of the beams are unrestrained. Pallet decks are attached to stringer or block spacers and therefore represent a range of connection stiffness or fixity. The effect of this connection fixity can be modeled using springs of appropriate stiffness and appropriate degrees of freedom and then by

summing the moments around critical locations in these connections. The critical location is the inside edge of the stringer or block. The solutions for a semi-rigid connection are shown in Figure

4. The moment M_A is based on a force exerted by the fastener and the distance “d” to the inside



Note: $\overline{2P}$ indicates the sum of the all reaction forces

$$F_N = ky$$

K = rotational modulus (lbs./in.)

y = the vertical displacement of the deck at the nail location (in.)

$$M_A = F_N \times d$$

F_N = force exerted by nails

d = distance between nail and stringer

Boundary Conditions

- (a) At $x_2 = 0$; $Q_0 = P$
- (b) At $x_1 = 0$ and $x_2 = 0$; $M(X_1) = M(X_2) = M_0$
- (c) At $X_1 = 0$ and $x_2 = 0$; $y(x_1) = y(x_2) = y_0$
- (d) At $x_2 = L_2$; $M=M_0$
- (e) At $x_2 = L_2$; $Q = -P$

Figure 4. Modification of the beam theory to reflect the stiffness or semi-rigid connections in pallets between deck boards and stringers.

edge of the stringer. This is determined from a measured joint rotation modulus. This method of modeling the stiffness of semi-rigid nailed connections in pallets has been validated by Samarasinghe (1987), among other researchers. In a similar way, models were developed assuming completely free ends and fixed ends. Free ends were unrestrained and fixed ends were represented by springs of infinite stiffness at the connections between the deck and stringer. The reader is referred to Yoo (2011) for the derivation of these relationships. This represents the full range of joint fixity that can occur in pallet connections. The model inputs are the elastic modulus (MOE) of the pallet deck, the joint rotation modulus, (K), for semi rigid connections, and the packaging stiffness (k). From the model predictions of packaging deformation along the length of the interface and the stiffness of the packaging, the force along the beam at the interface can be calculated using to Hook's Law. The stresses are assumed to vary symmetrically and in only one geometric dimension along the length of the pallet deck component.

Model Validation

Measurement of test specimen stiffness

To validate the model, a common pallet and packaging configuration was used and simulated. This included a corrugated container supported by a wood pallet section. Example pallet sections are shown in Figure 5. The deck spanning the stringer segments was 18 inches long and 3.5 inches wide.

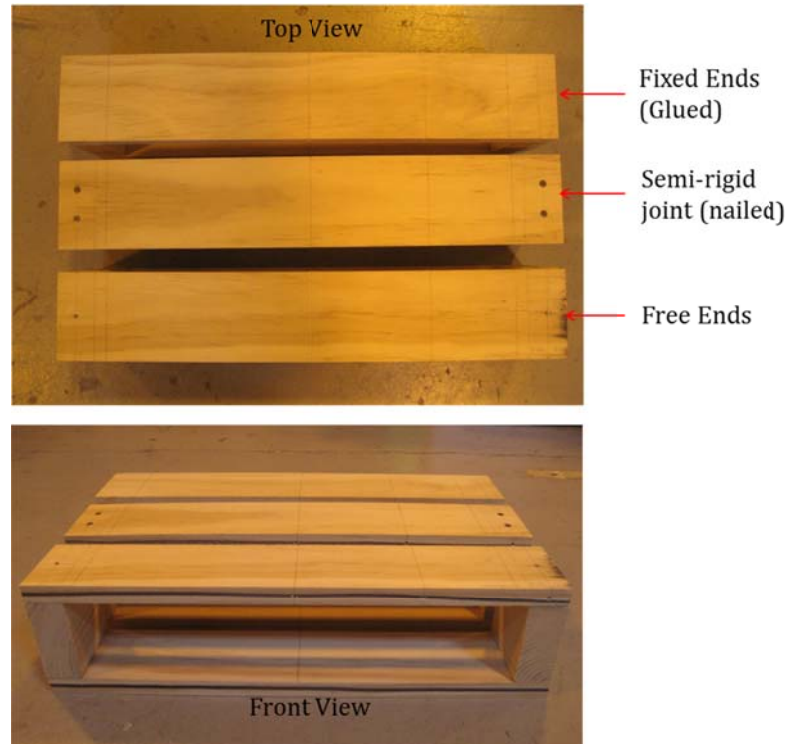


Figure 5. Photographs of the wood pallet, two stringer, sections, used for measuring compression stresses on packaging.

A range of pallet deck stiffness was represented by two different thicknesses (0.375 and 0.750 inches) of deck components and a range of elastic moduli. Three levels of joint fixity and two levels of semi rigid connection stiffnesses were also incorporated. The corrugated container was an RSC style, measuring 15.5 L x 7.75 W, and 10 D inches, externally. The board grade was 42, 33C, 42. The package stiffness varied in accordance with its contents, empty, bottles, or flower sacks. The contents of plastic bottles and flower sacks within the corrugated container are shown in Figure 6. The methods used to measure the stiffness of the pallet deck and packaging and the joint rotation modulus are shown in Figures 7, 8, and 9, respectively. This included a third point bending test of the pallet decks, a cantilever rotation test of the semi-rigid joints, and the vertical



Figure 6. Photograph of the opened test corrugated container with the full plastic bottles and small sacks of flour.

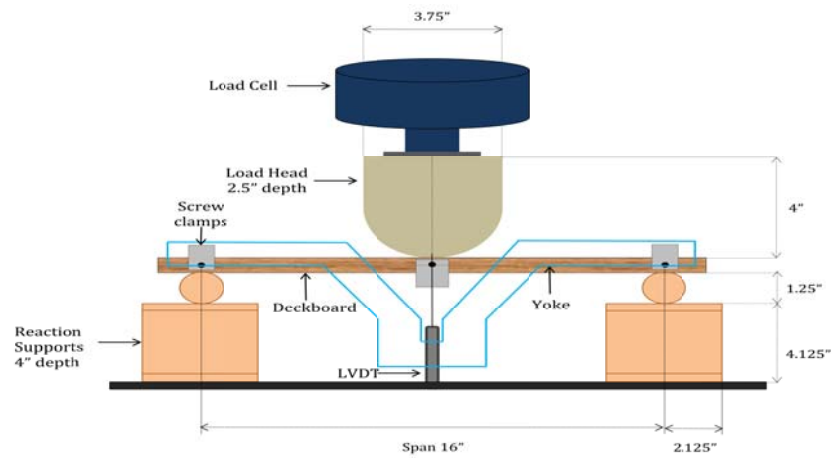


Figure 7. Schematic diagram of the test for measuring the modulus of elasticity of the pallet section decks.



Figure 8. A photograph of the test for measuring the compression stiffness of the corrugated container.

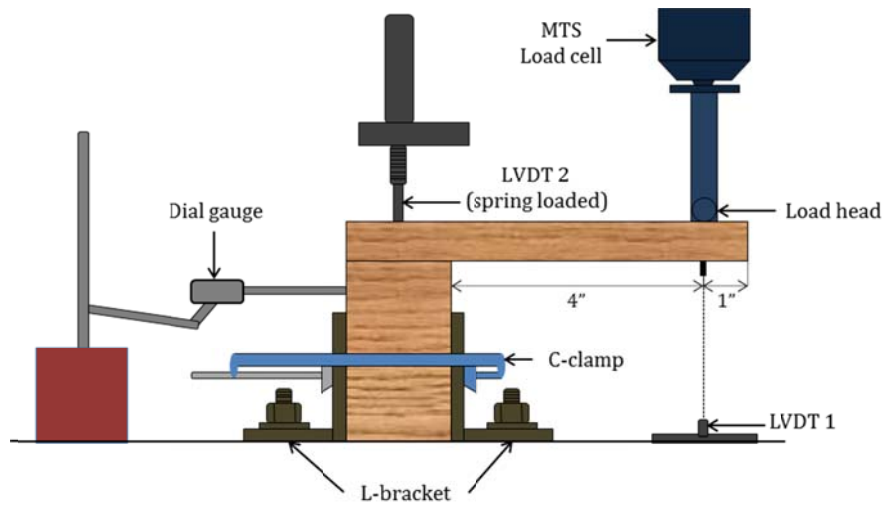


Figure 9. Schematic diagram showing the test for measuring the joint rotation modulus of the semi-rigid pallet connection.

compression of the corrugated container. The container stiffness was measured along the box length and included in the corner support. The average stiffness value was used as inputs into the model. These model inputs are summarized in Table 1. Three replicate rotation modulus and package stiffness tests were performed.

Table 1. Measured properties used as inputs into the compression stress model

	MOE (lbs./in. ²)		End Fixity	Rotation Modulus (in.-lbs./radian)		Packaging stiffness (lbs./in.)			
	3/4"	3/8"		3/4"	3/8"	Bottled box	Empty box	Flour sacks	
A-1	1340000	B-1	1081000	Free	6494	5921	1345	854	618
A-2	1236000	B-2	1077000	Free					
A-3	1246000	B-3	1082000	Free					
A-4	1118000	B-4	1023000	Semi					
A-5	1178000	B-5	1626000	Semi					
A-6	730700	B-6	1408000	Semi					
A-7	854500	B-7	716800	Fixed					
A-8	1238000	B-8	822800	Fixed					
A-9	1173000	B-9	1131000	Fixed					

Measurement of Compresses Stresses at the Pallet Deck and Package Interface

Two pressure sensor mats were used to directly measure the pressure between the pallet deck and the corrugated container. The sensitivity of the pressure sensors was 0-5 and 0-30 psi. A strain gage sensor mat is shown in Figure 10. (The sensel size was 0.25 inch square.) The test setup for measuring the compression stress at the interface between the package and the pallet deck is shown in Figure 11. The load was applied using a 10 kip servo-hydraulic MTS test machine to a level of 6 to 7 pounds per square inch.

Results

Figure 12 is a typical visual representation of the stress distribution between the packaging and pallet deck for both sensors, as a function of package and pallet deck stiffness.

The red denotes high stress and the black, zero stress. It is clear that the compression stresses are high over the stringer segments and very low between these areas of deck support.

Data within the sensitivity range of each of the two mats was then merged into one data set as shown in the example in Figure 13. Assuming symmetry, doubled the usable observations of the stress gradients. Figures 14, 15, and 16 are plots of the measured and predicted compression stresses for the three different connection fixities. The average applied stress during the test varied from 6.25 to 6.82 psi. With the exception of the free end, low stiffness pallet section, the agreement between predicted and measured is good. The poor correlation of the low stiffness pallet section is due to the lack of measured stress above the actual 24 psi limit of the sensors. The maximum predicted compression stresses were as high 50 psi over the pallet posts or stringers while packaging between the stringers is under negligible compression stress.

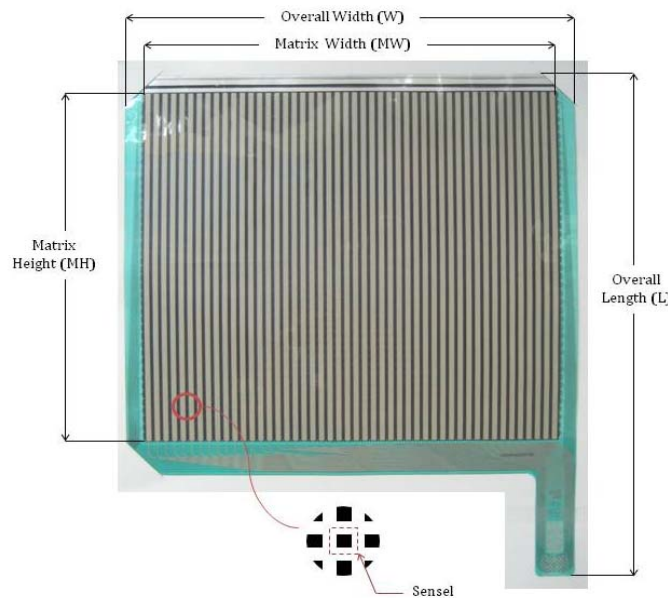


Figure 10. Photograph of the strain gage pressure sensor mat used to measure the pressure distribution between the package and the pallet deck (sensel dimension 0.25 x 0.25 inches).

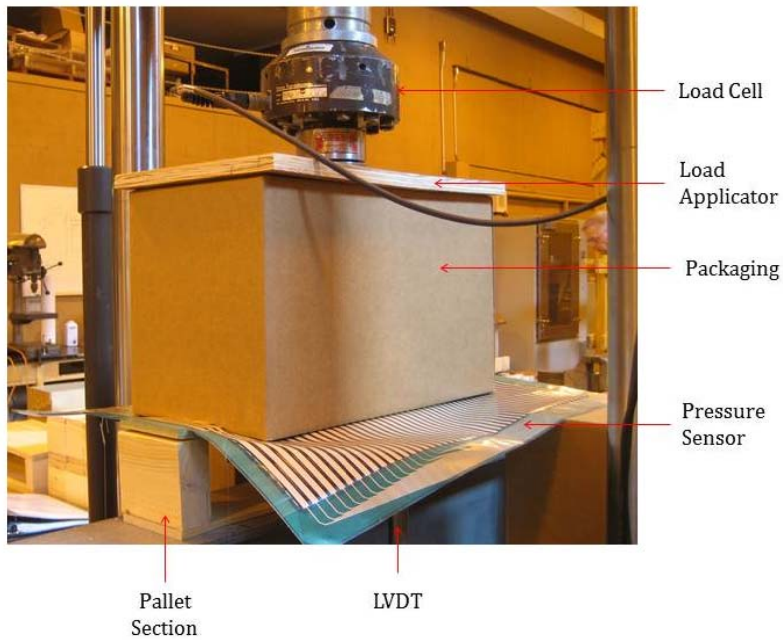


Figure 11. A photograph showing the test set-up for measuring the compression stress distribution between packaging and the deck of the pallet section.

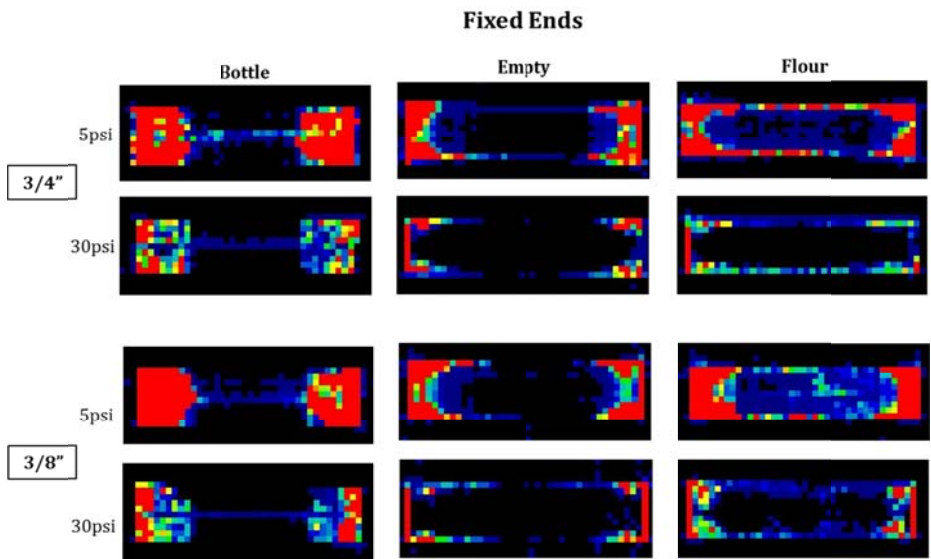


Figure 12. Visual representation of the measured compression stress distribution between the packaging and pallet deck.

Location (in.)	0.8	1.2	1.6	2	2.4	2.8	3.2	3.6	4	4.4	4.8	5.2	5.6	6	6.4	6.8	7.2	7.6	8	
0.4	24.094	22.491	19.802	21.493	21.294	7.688	20.299	10.819	0.000	0.525	0	0	0	0	0	0	0	0	0	0
0.8	24.094	11.011	24.094	24.094	22.092	3.547	18.515	23.092	13.131	1.095	0	0	0	0	0	0	0	0	0	0
1.2	16.938	8.914	24.094	24.094	24.094	0.000	13.325	24.094	14.102	1.503	0	0	0	0	0	0	0	0	0	0
1.6	24.094	10.819	16.545	3.907	0	0	7.220	16.933	15.858	0.000	0	0	0	0	0	0	0	0	0	0.616
2	16.152	9.293	3.1	1.331	2.107	1.607	10.723	24.094	22.992	0.000	0	0	0	0	0	0	0	0	0	0
2.4	24.094	17.627	7.033	0.000	9.959	0.000	3.727	8.725	19.604	5.641	0	0	0	1.044	3.624	0	0	1.642	0	0
2.8	22.491	24.094	21.693	5.273	8.630	9.388	24.094	11.972	0.000	0	0	0	0	0	0	0	0	0	0	0
3.2	19.010	24.094	23.392	3.727	0.000	0.000	16.250	12.454	9.769	0	0	0	0	0	0	0	0	0	0	0
3.6	24.094	24.094	24.094	5.549	8.630	4.088	15.467	11.395	23.994	0.74	0	0	0	0	0	0	0	0	0	0
Average	21.67	16.94	18.21	11.18	13.83	5.26	14.4	15.95	17.06	1.9	0	0	0	1.05	3.62	0	0	1.64	0.62	0

Location (in.)	8.4	8.8	9.2	9.6	10	10.4	10.8	11.2	11.6	12	12.4	12.8	13.2	13.6	14	14.4	14.8	15.2
0.4	0	0	0	0	0	0	0	1.713	0	4.633	3.727	3.100	0	0	0	0	3.100	0
0.8	0	0	0	0	0	0	0.511	1.641	12.164	22.691	21.393	24.094	22.192	24.094	22.791	14.394	20.696	11.298
1.2	0.996	0	0	0	0	0	0	6.288	4.451	0	8.536	8.820	6.474	24.094	20.895	24.094	5.181	
1.6	0	0	0	0	0	0	0	0	0	8.253	0	16.447	11.683	0	9.483	18.120	10.628	
2	0	0	0.787	0	0	0	0	8.914	12.261	0	12.937	9.769	14.783	12.841	19.604	24.094	11.683	
2.4	3.368	0	0.724	3.368	1.093	3.546	0	3.100	5.641	7.970	3.997	0	0	0.000	4.088	14.588	13.034	
2.8	0.819	0	0	0	0	0	0	0	9.483	3.458	5.826	0	0	7.126	11.972	21.593	8.914	
3.2	0	0	0	0	0	0	0	0	5.641	17.135	13.810	5.733	8.536	4.542	24.094	24.094	24.094	
3.6	0	0	0	0	0	0	0	0	7.688	22.092	20.695	15.761	24.094	24.094	24.094	24.094	24.094	9.578
Average	1.73	0	0.76	3.37	1.1	3.55	0.51	2.37	8.104	13.51	12.18	14.31	15.57	14.25	19.32	16.97	21.72	12.57

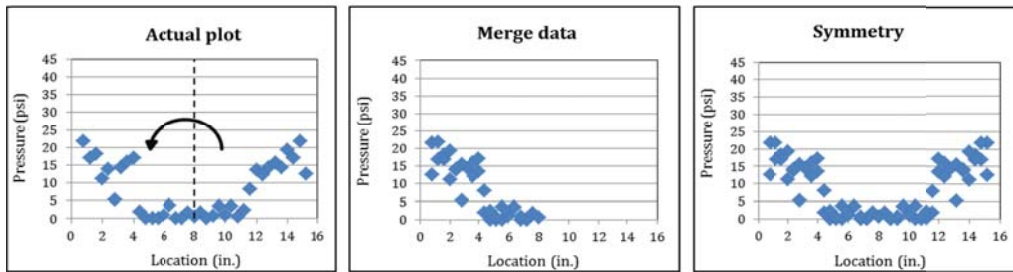


Figure 13. Typical measured compression stresses across the pallet deck after the data from both sensor mats was merged. Symmetry of stress across the deck was used to double the useful observations.

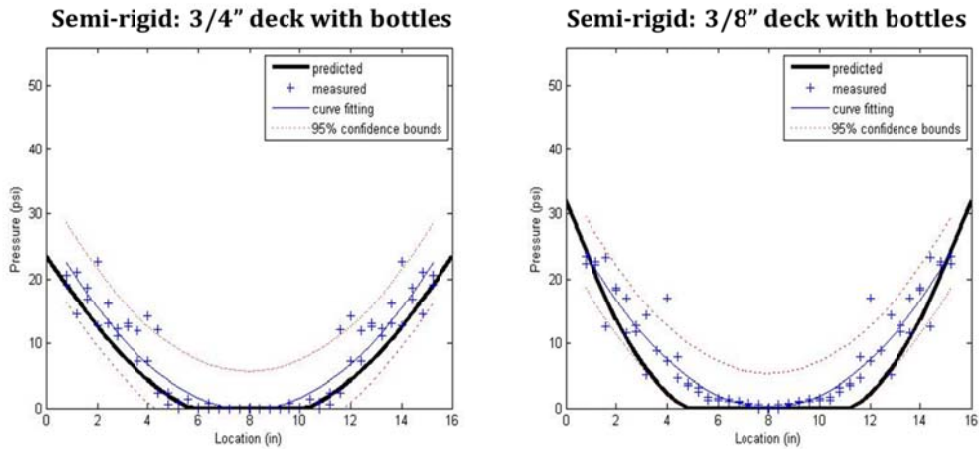


Figure 14. Correlation between measured and predicted compression stresses at the interface between a 16 inch long corrugated container containing plastic bottles and a wood pallet deckboard with semi-rigid connections to two stringers. Maximum compression stress is over the stringer and minimum compression stress in between stringers.

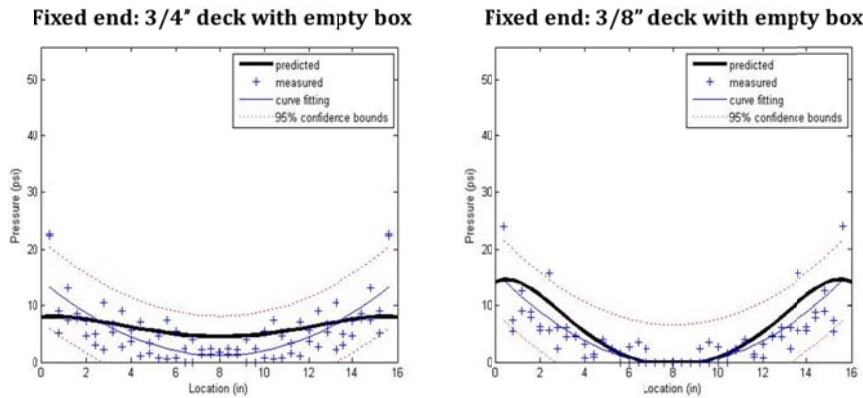


Figure 15. Correlation between measured and predicted compression stresses at the interface between a 16 inch long empty container and a wood pallet deckboard with fixed connections to two stringers. Maximum compression stress is over the stringer and minimum compression stress in between stringers.

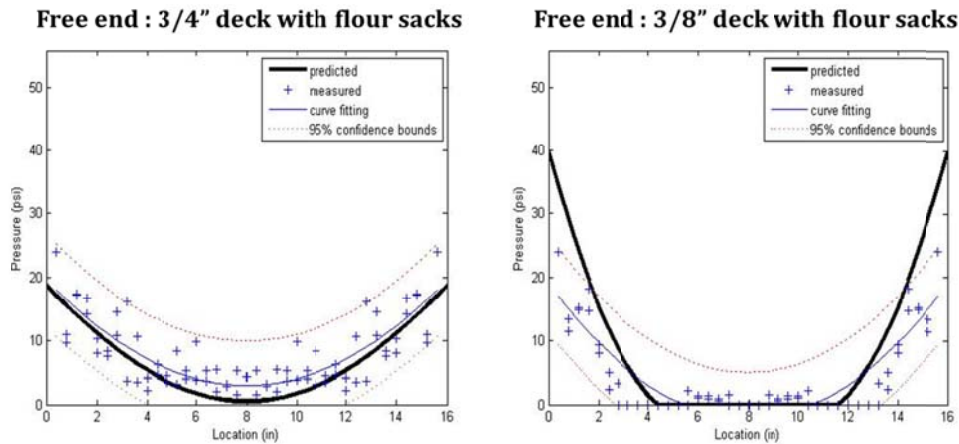


Figure 16. Correlation between measured and predicted compression stresses at the interface between a 16 inch long flour sacks and a wood pallet deckboard spanning two stringers without restraint. Maximum compression stress is over the stringer and minimum compression stress in between stringers.

The following trends are evident from both measured and predicted stresses. Compression stress on the packaging increases as the packaging stiffness increases. However, the compression stress is inversely related to the pallet deck stiffness. The connection fixity significantly influences pallet deck stiffness. According to these test results, the manipulation of the pallet deck stiffness can significantly alter the effective bearing area and consequently alter the maximum compression stress to which the packaged product is exposed when unitized and moved through supply chains. The maximum stresses measured in this study varied from about 10 to 25 psi. However, the maximum measured stress was limited by the sensitivity level of the sensors used. Weigel et al. 1999 showed that stiffer pallet decks also reduced the transmissibility of vibration accelerations during shipping. It is therefore increasingly clear that pallets with stiffer decks reduce the stresses to which packaging and product are exposed during the unit load portion of supply chains.

Conclusions

1. A modification of the principles of an elastic beam supported by an elastic foundation can be used to predict the compression stress distributions between pallet decks and packaged products. The required inputs are the compression stiffness of the packaged product and the bending stiffness of the pallet deck.
2. The compression stresses at the interface are not uniformly distributed across the pallet deck. Stress concentrations occur over pallet stringers and packaging.
3. Average applied compression stress levels of 6-7 psi resulted in compression stresses on packaging from 0 to over 50 psi for the specimens tested. The compression stress amplification or intensity can be as high as five to ten times the average applied stress.
4. Stiffer pallet decks and stiffer connections between the pallet deck and stringers or blocks, significantly reduces the maximum compression stress on the packaged product.
5. Pallet design can be used to reduce the compression stresses on packaging and reduce packaging cost.

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